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ABSTRACT

Performance objectives are stated for this secondary school instructional unit concerned with introduction to the stretcher and shrinker approach, solution of simple equations, factoring composite numbers into primes, definition of prime numbers, and communication skills with computational concepts. The course of study is intended for students having competence in the basic computational skills with whole numbers. Comments are presented concerning teaching of the course. Included are a time schedule for instruction of stretching machines; an outline of the topics and objectives included in the course content; suggestions for administration of pre- and posttests; and lists of classroom supplies, teaching aids, and state-adopted and other texts for enrichment and practice purposes. (CC)

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AUTHORIZED COURSE OF INSTRUCTION FOR THE



INTRODUCTION TO STRETCHING MACHINES

5211.08

5212.08

MATHEMATICS

DIVISION OF INSTRUCTION • 1971

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QUINMESTER MATHEMATICS  
COURSE OF STUDY  
FOR  
INTRODUCTION TO STRETCHING MACHINES  
5211.08  
5212.08

(EXPERIMENTAL)

DIVISION OF INSTRUCTION  
Dade County Public Schools  
Miami, Florida 33132  
1971-72

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## PREFACE

The following course of study has been designed to set a minimum standard for student performance after exposure to the material described and to specify sources which can be the basis for the planning of daily activities by the teacher.

The course sequence is suggested as a guide; an individual teacher should feel free to rearrange the sequence whenever other alternatives seem more desirable. Since the course content represents a minimum, a teacher should feel free to add to the content specified.

Any comments and/or suggestions which will help to improve the existing curriculum will be appreciated. Please direct your remarks to the Consultant for Mathematics.

All courses of study have been edited by a subcommittee of the Mathematics Advisory Committee.

## CATALOGUE DESCRIPTION

The first of four quins designed to develop computational skills with fractions, percents, and decimals using the "stretcher and shrinker" approach developed by the University of Illinois Committee on School Mathematics. Includes factoring, equations, primes, exponents, and rearrangement (commutativity).

Designed for the student who has competence in the basic computational skills with whole numbers.

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### GOALS

1. To introduce the student to the stretcher and shrinker approach
2. To give the student positive, success-oriented experiences in math and increase his motivation
3. To develop and increase the student's understanding of, and skill with, the concepts of multiplication, factoring primes, exponents, equations and rearrangement (commutativity)
4. To increase the student's communication skills with computational concepts

### PERFORMANCE OBJECTIVES

The student will be able to:

1. Solve simple equations involving stretching machines.
2. Given non-prime and prime numbers, state at least one other name using the process of multiplication for that number, where possible.
3. Solve simple word problems and puzzles utilizing multiplication and factoring.
4. Factor a composite number less than 100 into primes and write the number in exponential notation.
5. Explain and give examples of the Rearrangement Principle (Commutativity).
6. Define prime numbers and list the first 20 primes.
7. Recall whole number multiplication facts with factors 1 - 10.

## STRATEGIES

Before beginning this quin, the teacher should read carefully the course description and comments contained in the Teacher's Edition and the Activities Handbook. In addition to these, the following general comments apply:

- (1) Entering competencies required include: addition, subtraction, multiplication of one and two-digit numbers, and division of whole numbers by one and two-digit divisors. It is recommended that a pretest be given and item-analyzed. The Skills Mastery Test contained in the GOALS book Dade County Public Schools Bulletin No. 7-H is recommended. If the answers are placed on computer cards, the services of CDP (teacher-made text program) can be used for more complete test analysis. Retain these test results to compare with the posttest for 5211.20/5212.20 - Decimals to assess student progress. This text can provide the pretest data for the four-quin series.
- (2) In order to maintain student interest it is recommended that the teacher stick closely to the story line. A machine either stretches or shrinks, not multiplies or divides. The kids will realize the multiplier and divider properties of the machines and will verbalize this. You should acknowledge the correctness of this and then go on using machine terminology. One of the major benefits of this course is its novelty, don't destroy it.
- (3) The heart of the course is found in the hundreds of activities found in the Activities Handbook, activities which foreshadow, expand, drill and supplement the development found in the text. The frequent and short quizzes found in that handbook help students to gain confidence, while at the same time reviewing small blocks of pages.
- (4) It is practically impossible to use all the activities in the handbook. The activities are divided into required and optional. Plan to use all the required activities and those optional activities that are appropriate to your class and the time available.
- (5) It is recommended that the overhead projector be used extensively. Many of the activities utilize transparencies, and transparencies of important workbook pages should be made to assist with discussion. Transparencies of quizzes and tests are valuable so that students can either grade each other's paper or immediate feedback and discussion opportunities can be provided.



- (6) The work text should be used primarily in a class-discussion approach, hopefully utilizing the overhead projector. Too much can be lost by simply working pages. The important ideas should be expanded and emphasized.
- (7) Discretion should be used in deciding whether or not to allow books to go home. It has been found effective to have row leaders distribute and collect books each day, leaving the books in the room. A beginning activity on the board can be used to expand on and review the material, and to settle the students while the row leaders are distributing the books.
- (8) The course was designed to be used in a work text type program. Much thought and planning should occur before utilizing the text as nonconsumable materials.
- (9) Classroom supplies: Stretchers and Shrinkers

Assuming a class size of 30, the following supplies would be consumed by the class in one year. (4 quins: 5211.08/5212.08-5211.20/5212.20.

- 30 - student sets (Book 1-4) of Stretchers and Shrinkers (if used consumably)
- 200 - index cards (3x5)
- 10 - reams of duplicator paper

The materials which follow may be used with several classes and should be saved from year to year:

- 1 - Teacher's Edition of Stretchers and Shrinkers
- 1 - Activities Handbook for Stretchers and Shrinkers
- 1 - meter stick
- 1 - yard stick
- 1 - box colored chalk
- 2 - boxes (100 sheets/box) thermal spirit masters
- 2 - boxes (100 sheets/box) transparency film for overhead projector
- 30 - scissors
- 30 - rulers (marked in inches and centimeters)
- 200 - file folders

You will also require the daily use of an overhead projector (with colored pens) and a file cabinet (at least three drawers).

## STRETCHERS AND SHRINKERS

### Time Schedule

#### Introduction to Stretching Machines

##### Chapter 1

1-30	Stretching Machines 2 Quizzes	9 days
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##### Chapter 2

31-54	Hookups and Factoring 2 Quizzes	7 days
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##### Chapter 3

55-82	Factoring, Factors and Equations 2 Quizzes	8 days
	Test	

##### Chapter 4

83-126	Punch Cards, Factoring Primes 3 Quizzes	11 days
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##### Chapter 5

127-144	Repeater Machines Exponents	5 days
	Test	
		<hr/> 40

If you are able to stick to this time schedule, you may use additional days for enrichment activities and computational review (see page

BOOK I, CHAPTER I: STRETCHING MACHINES

TEXT PAGES	TOPIC	OBJECTIVE
1-4	Labeling sticks according to length	<p>Given a collection of sticks of various lengths, the student is able to sort and label the sticks having the same length.</p> <p>Note: to fulfill this objective it is permissible for students to compare lengths with a card, a ruler, or any other measuring process they may devise.</p>
5-7 145-146	Comparing lengths of sticks	<p>a. Given the lengths of two sticks, expressed in the same unit, the student is able to tell which is longer, shorter.</p> <p>b. Given a unit of length (shown by a segment) and another length (shown by another segment), the student is able to express (tag) the second length in terms of the unit length.</p> <p>c. Given a unit of length, and the tag for a second length, the student is able to draw the second length. (The student may be given a grid for this.)</p>
8-9	Using stretching machines on sticks	Introductory -- no performance expected.
10-16	Establishing the relationship between input, machine, and output	<p>a. Given an input length, a stretching machine, and a grid, the student can draw and label the corresponding output length.</p>

(BOOK I, CHAPTER I: STRETCHING MACHINES)

TEXT PAGES	TOPIC	OBJECTIVE
10-16 continued		<p>b. Given a machine, a grid, and an output length, the student can draw and label the corresponding input length.</p> <p>c. Given an input length and an output length, the student can tell the corresponding stretching machine.</p>
17-20 147	Using tabular arrays for input, machine, and output problems	<p>Given:</p> <p>a. an input length, a machine, <u>no</u> grid,</p> <p>b. a machine, an output length, <u>no</u> grid,</p> <p>c. an input length, an output length,</p> <p>the student will tell the corresponding</p> <p>a. output length</p> <p>b. input length</p> <p>c. stretching machine</p> <p>Note: To meet this objective it is appropriate for students to transfer the given information to a grid in order to construct the answer.</p>

(BOOK I, CHAPTER I: STRETCHING MACHINES)

TEXT PAGES	TOPIC	OBJECTIVE
21-27	Establishing notational conventions for depicting input, machine, and output relationships on history cards and output tags	Given: a. paired history cards and output tags for specified sticks, the student can identify correct and incorrect pairings. b. pairings of incomplete history cards and output tags, the student can compute the missing data.
28	Establishing notational conventions for input, machine, output relationships in equation form	Given an equation in which one of the three critical variables is missing, the student can compute the missing data.
29	Establishing conventions for interpreting worded statements about inputs, machines, and outputs, and equations which describe the relationships	Given worded statements and their corresponding equations, students can make appropriate matchings.

(BOOK I, CHAPTER I: STRETCHING MACHINES)

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TEXT PAGES	TOPIC	OBJECTIVE
30	CHAPTER I SUMMARY	<p>a. By the completion of Chapter I, the student should be able to solve INPUT-MACHINE-OUTPUT exercises like those described in the objectives for pp. 28-29, but he need not be able to solve such problems by immediate recall of a multiplication fact. He may be permitted to construct solutions using grids, card measuring, skip counting, finger counting, etc.</p> <p>b. Students should be able to give the output of a 1-machine, for <u>any</u> whole number input.</p> <p>c. Students should be able to recognize the use of the 1-machine when the input and output lengths are the same.</p>

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BOOK I, CHAPTER 2: HOOKUPS AND FACTORING

TEXT PAGES	TOPIC	OBJECTIVE
31-34	Introducing hookups and appropriate terminology for intermediate outputs	Given an input and a hookup of stretchers on a grid, the student can draw the intermediate and final output sticks.
35-37	Using intermediate sticks	<ol style="list-style-type: none"> <li>Given an input length, and a hookup of machines, the student can supply the intermediate output and the final output lengths.</li> <li>Given the input length, the intermediate output, and the final output, the student can supply the hookup of machines to do the job.</li> <li>Given the input and output lengths, and one machine of a hookup, the student can supply the intermediate length and the other machine of the hookup.</li> </ol>
38-41	Establishing notational conventions for input, hookup, output relationships on history cards and output tags	Similar objectives as for pp. 35-37 except information is given in "history card" format.
42-43 149-150	Establishing notational conventions for input, hookup, output relationships in equation form	Similar objectives as for pp. 35-37 except information is given in equation format.

(BOOK 1, CHAPTER 2: HOOKUPS AND FACTORING)

TEXT PAGES	TOPIC	OBJECTIVE
44-50	Finding different ways of doing a job	<ul style="list-style-type: none"><li>a. Given nonprime and prime machines, the student can give, when possible, at least one hookup (no 1-machines) that will do the same job.</li><li>b. Given a job, the student can give either a single machine or at least one hookup that does the same job.</li></ul>
51-52	Writing equations to tell different ways of doing a job	Same objectives as for pp. 44-50 except information is given in equation format.
53-54	Introducing the terminology of factoring and differentiating multiplying from factoring	Given a hookup of machines, the student can, by multiplying, find the equivalent single machine, and, given a single machine, can by factoring find an equivalent hookup.



BOOK 1, CHAPTER 3: FACTORING, FACTORS, AND EQUATIONS

TEXT PAGES	TOPIC	OBJECTIVE
55-56	Doing jobs at the Zabbranchburg Factory	Introductory -- extending the domain of machines to all non-zero whole numbers.
57-59	Establishing notational conventions for factorizations on routing slips	Introductory -- Given a routing slip, the student can indicate with a factoring diagram an alternative hookup to use when a desired machine is broken.
60-63	Using factoring diagrams for can and cannot-do jobs	Practice -- Given a job, the student can use a factoring diagram to indicate, when possible, alternative ways of doing the job.
64-67	Defining and finding factors	<ol style="list-style-type: none"><li>Given a machine or job, and one of its factors, the student can supply the other factor.</li><li>Given a machine or job, and a machine which is not a factor, the student recognizes that no hookup containing that machine will do the job.</li><li>Given a number less than 15, the student can give all of its factors.</li></ol>

(BOOK 1, CHAPTER 3: FACTORING, FACTORS, AND EQUATIONS)

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68-74 151-152	Introducing "equation related" terminology and solving equations	Given a hookup equation, the student can supply a missing factor, or if there is no solution, can recognize this fact.
75-78	Extending the domain of inputs and outputs to magnitudes other than length	Given an equation (magnitudes other than stick lengths), the student can solve the equation.
79-82	Using equations to solve problems	Given a problem, the student can use the stretcher machine model to find the solution, and use bar graphs to organize and interpret data.

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BOOK 1, CHAPTER 4: PUNCH CARDS, FACTORING, AND PRIMES

TEXT PAGES	TOPIC	OBJECTIVE
83-84 153-154	Introducing and stating the re-arrangement principle	Given hookups of machines, the student can use the re-arrangement principle to tell if they are equivalent re-arrangements.
85-92	Using the rearrangement principle to establish notational conventions for jobs or hookups on punch cards	Given specified jobs, the student can (using punch card) show various ways of doing the job.
93-94	Using punch cards and the rearrangement principle	Given two hookups, one a rearrangement of the other, the student recognizes that they do the same job.
95-101	Punching cards to show factorizations	<p>a. Given a job, the student can give a hookup that will do the job and correctly complete a punch card to show how the job can be done.</p> <p>b. Given a punched card, the student can tell what job is shown.</p>
102-108 155	Deciding whether jobs are even or odd	a. Given a hookup, the student can tell whether it does odd or even jobs.

BOOK 1, CHAPTER 4: PUNCH CARDS, FACTORING, AND PRIMES

TEXT PAGES	TOPIC	OBJECTIVES
102-108 - continued 155		b. Given a hookup, one of the machines hidden, the student can tell whether the hookup does odd or even jobs or it is impossible to tell.
109-114 156	Getting rid of machines that Anabru doesn't need	Exploratory and introductory.
115-116 157	Defining primes	
117-120	Using primes in hookups and factoring diagrams	a. Given a job, the student can construct a hookup of prime machines.  b. Given a hookup, the student can tell if it's a hookup of primes and find the single machine equivalent to the hookup.  c. Given a machine, the student can factor it into primes and report the results in a factoring diagram.

(BOOK 1, CHAPTER 4: PUNCH CARDS, FACTORING, AND PRIMES)

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TEXT PAGES	TOPIC	OBJECTIVE
121-124	Using Anabru punch cards to depict prime factorizations	Given a machine (a job), the student can show on the Anabru punch card the prime factorization for the given machine and, conversely, given the prime factorization for a given job shown on the Anabru punch card, the student can give the job specified (single machine).
125-126	Writing equations for prime factorizations of jobs	Given a machine or job, the student can write an equation showing the prime factorization for the given job.

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BOOK 1, CHAPTER 5: REPEATER MACHINES AND EXPONENTS

TEXT PAGES	TOPIC	OBJECTIVE
127-129	Introducing repeater machines	Given a hookup of repeated machines, the student can express the hookup in repeater machine form using a base and exponent.
130-134	Using repeater machines	<p>a. Given a job, the student can use the appropriate repeater machine to do the job.</p> <p>b. Given a repeater machine, the student can find the job the machine can perform.</p> <p>c. Given a hookup of repeaters, the student can find the job the hookup can do.</p>
135-138	Introducing and using vocabulary [base(s), exponent(s), and power(s)]	Given a base and an exponent, the student can tell what job the repeater will do.
139-142 160	Hooking up repeater machines	Given a hookup of repeaters or non-repeaters, the student can find the job the hookup will do or a simpler hookup of repeaters to do the same job.

(BOOK 1, CHAPTER 5: REPEATER MACHINES AND EXPONENTS)

TEXT PAGES	TOPIC	OBJECTIVE
143-144	Using exponents in prime factorizations	Given a job (composite number), the student can factor it into primes, and then express the result in exponent notation.
BOOK 1	Summary	<p>By the end of Book 1, it is desirable that the student acquire the following skills:</p> <ul style="list-style-type: none"><li>a. Recall whole number multiplication facts with factors 1-10.</li><li>b. Factor a composite number less than 100 into primes.</li></ul>

### PRETEST

For the pretest use the Skills Mastery Test contained in the Goals book Dade County Public Schools Bulletin No. 7-H or a test that is similar.

It is recommended that the students bubble their answers on IBM answer cards and the services of Central Data Processing be utilized to provide complete test analysis.

### POSTTEST

Utilize Activity 47 (Test 1) and Activity 76 (Test 2) in the Activities Handbook as the posttest for this Quin.

#### Suggested Sources of Enrichment and Practice Activities:

##### A. State adopted

1. Crouch, William H. Coordinated Cross Number Puzzles A, B, C. New York: McCormick-Mathers Publishing Co., 1970.
2. Denholm, R. A. and Blank, V. D. Mathematics Structure and Skills 1st Book. Chicago: Science Research Associates, 1968.
3. Foley, Jack; Jacobs, Wayne and Basten, Elizabeth. Individualizing Mathematics. Menlo Park, California: Addison Wesley Publishing Co., 1970.

#### Skills and Patterns

Whole Numbers  
Numbers-Patterns-Theory  
Sets  
Fractions—Addition and Subtraction  
Fractions—Multiplication and Division  
Decimals--Meanings and Operations

4. Johnson, D. A., et al. Activities in Mathematics: First Course: Number-Patterns. Glenview, Illinois: Scott, Foresman and Co., 1971.



5. Sobel, Max A., et al. Essentials of Mathematics Series: Book 1. Boston: Ginn and Company, 1970.
6. Tucker and Wheeler. Mathematics Laboratory. New York: McCormick-Mathers Publishing Co., 1970.
7. Wirtz, Robert W., et al. Math Workshop Levels C, D, E. Chicago: Encyclopedia Britannica Educational Corp., 1964.

B. Non-state adopted

1. Brandes, Louis G. Yes, Math Can Be Fun. Portland, Maine: J. Weston Walch, 1960.
2. Dumas, Enoch. Arithmetic Games. Palo Alto, California: Fearon Publishers, Inc., 1960.
3. \_\_\_\_\_. A Collection of Cross Number Puzzles.
4. Larsen, Harold D. Games to Play.
5. \_\_\_\_\_. Guzintas.
6. \_\_\_\_\_. Ways to Multiply
7. \_\_\_\_\_. Brain Teasers  
Evanston, Illinois: Harper and Row, Publishers, 1961.
8. Meyer, Jerome S. "Arithmetricks." Englewood Cliffs, N. J.: Scholastic Magazine, 1965.
9. Wagner, Guy, et al. Arithmetic Games and Activities. Darien, Connecticut, 1964.